

# Real-coded genetic algorithm for Bragg grating parameter synthesis

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We propose to use a genetic algorithm to determine the physical parameters of Bragg gratings from their reflection spectra for both design purposes and fiber sensor applications. A real-coded genetic algorithm is used for inversion purposes, along with an  $F$ -matrix formalism for synthesis of uniform, chirped, and apodized gratings. An example of bandpass filter design is also studied. The method is easily applicable and shows promising results. © 2001 Optical Society of America

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## I. INTRODUCTION

Fiber Bragg gratings are an essential part of fiber-optic communication. They are used in wavelength division multiplexing (WDM) to add or remove wavelengths to optical channels<sup>1</sup> or are used as filters. Bragg gratings have other important applications, e.g., as strain and temperature sensors<sup>2,3</sup> and as laser source tuners.<sup>4</sup>

The synthesis of Bragg gratings that conform to a particular spectrum has no exact direct methodology. Iterative methods must be used to solve the problem, as no direct inverse equations can be obtained. Some methods have been proposed: Winick and Roman<sup>5</sup> used an inverse Fourier transform. This method is termed the first-order Born approximation, but it works only for gratings with low reflectivity.

One inverse solution was discovered by Song and Shin,<sup>6</sup> who solved the Gel'fand–Levitan–Marchenko equations in quantum mechanics. Their method works well, but it requires one to express the reflection coefficients as rational expressions. Peral *et al.*<sup>7</sup> expanded on the previous method and obtained an iterative solution to the Gel'fand–Levitan–Marchenko equations. Their method was relatively quick and gave good results, even at high reflectivity. Feded *et al.*<sup>8</sup> proposed a differential layer-peeling method that takes into account all the multiple reflections inside the grating, yielding an exact inverse scattering method. The method gives good results and shows promise.

These methods of grating synthesis provide good results for solving specific problems, but they are not fully useful as design tools. The previous solutions do not permit one to weigh certain parts of the spectrum more than others. Certainly, in design it is sometimes desirable to have characteristics that are more important than others. An algorithm that permits weighting would therefore have more design applications.

Skaar and Risvik<sup>9</sup> used a binary-coded genetic algo-

riithm along with a Runge–Kutta algorithm to design filters with specific characteristics. Their algorithm fitted the desired spectrum well and permitted the weighting of more important parts of the design.

In this paper we describe the use of a real-coded genetic algorithm in conjunction with the  $F$ -matrix formalism for Bragg gratings.<sup>10</sup> The objective of this study can thus be stated as follows: Given the reflection spectrum of a Bragg grating (and only the reflection spectrum, i.e., no phase information), determine the design variables (length of grating  $L$ , grating period  $\Lambda$ , and difference in refractive indices  $\Delta n$ ) that would produce such a spectrum.

## 2. GENETIC ALGORITHM

The genetic algorithm was proposed by Holland.<sup>11</sup> He proposed a binary-coded algorithm modeled on Darwinian principles of survival of the fittest, with a random but structured exchange of information. A random population of individuals, or solutions to the problem, is created, and in turn the parameters of these solutions are combined and mutated to produce other, it is hoped better, solutions. This process is repeated for a number of generations until the desired solution is obtained. The basic genetic algorithm has three procedures: selection, reproduction, and mutation. Genetic algorithms have been used to solve a variety of nonlinear problems,<sup>12–14</sup> and their strengths and weaknesses are well documented.

An explanation of binary-coded algorithms, whose chromosomes comprise 0's and 1's, is provided in Ref. 9, and the reader is encouraged to peruse that text for further information. Real-coded genetic algorithms, whose chromosomes comprise real numbers, were shown to give better performance<sup>15</sup> than binary-coded genetic algorithms, and their implementation in this study is explained in the following sections.

### A. Initial Population

The initial population is made up of randomly determined parameters within specified boundaries. These parameters are called the genes of the chromosome, or of an individual.<sup>16</sup> For example, a population  $P$  of three random individuals ( $I_1, I_2, I_3$ ), with genes that represent the values of three design variables  $L$ ,  $\Lambda$ , and  $\Delta n$ , could be

Component of $P$	$L$	$\Lambda$	$\Delta n$
$I_1$	6.5	543.02	0.02
$I_2$	10.32	536.82	0.0006
$I_3$	8.70	545.76	0.0098

### B. Selection

One selects individuals by evaluating the performance of each one and then ranking them from best to worst. It is in this process that we choose which individuals will be used for reproduction. Many selection criteria exist, but the one used here was developed by Davis<sup>17</sup> and is explained below.

The performance of the individuals, in this case, is the difference between the measured (or desired) value of the spectrum and the calculated value. This function, often called the objective function, is

$$F = \sum_{\lambda} (R_m - R_c)^2, \quad (1)$$

where  $R_m$  is the measured (or desired) value of the reflection and  $R_c$  is the computed value for a particular wavelength at each wavelength. Here the value of  $F$  is to be minimized, as we wish to minimize the difference between the desired value and the calculated value. Once each individual's performance has been determined, the rating system proposed by Davis<sup>17</sup> is used. The individual with the best performance receives a relative weight (RW) of  $\eta^a$ , where  $\eta$  is the number of individuals in the population and  $a$  is a weight that is usually 1.0–1.5. The second-best individual receives a RW of  $(\eta - 1)^a$ , and so on, until the worst individual receives a RW of 1.

The probability of reproduction of each individual is then calculated from the following formula:

$$\text{PR}_j = \frac{\text{RW}_j}{(1/\eta)\sum \text{RW}}, \quad (2)$$

where  $\text{PR}_j$  is the probability of reproduction for individual  $j$  (where  $j = 0, 1, \dots, \eta$ ) and  $\eta$  is, again, the total number of individuals in the population. We see that individuals with higher RWs have a better chance of reproducing.

One then uses a stochastic remainder selection procedure<sup>16</sup> to determine the frequency of each individual's selection (once selected, the individuals are inserted into a reproduction pool). For example, an individual with a PR of 2.1 will be selected twice for reproduction, whereas an individual with a PR of 1.8 will be selected once. The rest of the reproduction pool comprises the sorted fractional parts.

### C. Reproduction

Reproduction involves randomly selecting two parents from the reproduction pool. These parents are then crossed to create offspring. We use Wright's technique<sup>15</sup> to create three offspring and select the best two. From two parents,  $P_1$  and  $P_2$ , three offspring,  $O_1$ ,  $O_2$ , and  $O_3$  are created:

$$P_1, P_2 = \begin{cases} O_1 = 0.5P_1 + 0.5P_2 \\ O_2 = 1.5P_1 - 0.5P_2 \\ O_3 = -0.5P_1 + 1.5P_2 \end{cases}, \quad (3)$$

where the algebraic operations in Eq. (3) are performed on each parameter. In theory, these offspring should have inherited the best characteristics of each parent, and their solution should be better than that of the parents. This scheme has been proved to converge slowly toward the best solution.<sup>17</sup>

For example, if two parents  $P_1$  and  $P_2$  are composed of values ( $L, \Lambda, \Delta n$ ) of (10.2 mm, 540.3 nm, 0.00018) and (9.78 mm, 549.7 nm, 0.00008), respectively, their three offspring will have the following values:

$L$	$\Lambda$	$\Delta n$
9.99	545.0	0.00009
10.41	535.6	0.00023
9.57	554.4	0.00003

The performance of each offspring is evaluated, and the best two are kept to maintain a constant population size.

### D. Mutation

Mutation permits the introduction of extra variability into the population. Here we use Michalewicz's nonuniform mutation.<sup>18</sup> If a mutation occurs (according to the probability of mutation,  $p_m$ ), the change to a randomly selected parameter  $X$  is determined by the flip of an unbiased coin:

$$X_i = \begin{cases} X_i + \delta[\text{Max}(X_i) - X_i] & \text{(heads)} \\ X_i - \delta[X_i - \text{Min}(X_i)] & \text{(tails)} \end{cases}, \quad (4)$$

where the function  $\delta$  is

$$\delta(y) = yr(1 - t/T)^B, \quad (5)$$

in which  $r$  is a uniformly distributed random number from 0 to 1,  $t$  is the current generation,  $T$  is the maximum number of generations, and  $B$  is a weight exponent from 1.0 to 5.0. As can be seen from Eq. (5),  $\delta$ , the amplitude of the change diminishes as one approaches the maximum number of generations; hence the name "nonuniform." As we approach the maximum number of generations, the population should be close to the desired solution, and we do not wish to introduce too great a change in the individual(s).

Once a new generation is produced, the previous generation is eliminated. The process of selection, crossover, and mutation is repeated until a maximum number of generations is reached or until the objective function has reached a preset value.

### E. Elitism

Elitism is the process of ensuring that the best individual survives. If, in the process of creating offspring and once we have depleted the reproduction pool, the best individual is not better than the best individual of the preceding generation (the elite), then the elite is copied and replaces the best individual of the current generation. Elitism was used in the research reported below.

### 3. F-MATRIX FORMALISM

We use  $F$ -matrix formalism<sup>10</sup> to calculate the Bragg grating reflection spectrum. For a Bragg grating of length  $L$  and uniform period  $\Lambda$  as given in Fig. 1, the forward- and backward-propagating electrical fields can be given by

$$\begin{aligned} E_a(z,t) &= A(z)\exp[i(\omega t + \beta z)], \\ E_b(z,t) &= B(z)\exp[i(\omega t - \beta z)], \end{aligned} \quad (6)$$

where  $\beta$  is the propagation constant. The complex amplitudes follow the coupled-mode equations

$$\begin{aligned} \frac{dA(z)}{dz} &= i\kappa B(z)\exp[-i2(\Delta\beta)z], \\ \frac{dB(z)}{dz} &= -i\bar{\kappa}A(z)\exp[i2(\Delta\beta)z], \end{aligned} \quad (7)$$

where  $\Delta\beta = \beta - \beta_0 = 2n_{\text{av}}\pi/\lambda - \pi/\Lambda$  is the differential propagation constant related to the decoupling from Bragg condition  $\beta = \beta_0$  and  $n_{\text{av}}$  is the average (geometric) refractive index. Coupling coefficient  $\kappa$  for TE waves is given by

$$\kappa = \frac{i(1 - \cos m\pi)}{2m\lambda \cos \theta} \frac{\sqrt{2(n_2^2 - n_1^2)}}{\sqrt{n_2^2 + n_1^2}}, \quad (8)$$

where  $\theta$  is the incidence angle relative to the  $z$  axis and  $m$  is the mode number (1 for monomode fibers). If we suppose that there are forward and backward inputs into the grating, with the boundary conditions  $B(0) = B_0$  and  $A(L) = A_L$ , we can solve Eqs. (7) and obtain the closed-form solutions  $a(z) = A(z)\exp(i\beta z)$  and  $b(z) = B(z)\exp(-i\beta z)$ . We can now express the backward output (reflected wave)  $a_0$  and the forward output (transmitted wave)  $b_0$  by a scattering matrix:

$$\begin{bmatrix} a(0) \\ b(L) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a(L) \\ b(0) \end{bmatrix}, \quad (9)$$

where  $a(L) = A_1 \exp(i\beta L)$ ,  $b(0) = B_0$ , and

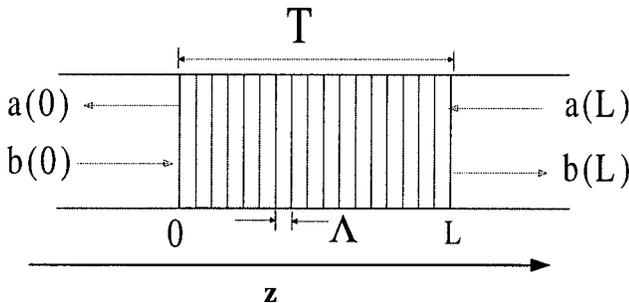


Fig. 1. Bragg grating model.

$$S_{11} = S_{22} = \frac{is \exp(-i\beta_0 l)}{-\Delta\beta \sinh(sl) + is \cosh(sl)}, \quad (10)$$

$$\begin{aligned} S_{12} &= \frac{k}{\bar{k}} S_{21} \exp(2i\beta_0 l) \\ &= \frac{k \sinh(sl)}{-\Delta\beta \sinh(sl) + is \cosh(sl)}, \end{aligned} \quad (11)$$

where

$$s = [(|\kappa|^2 - (\Delta\beta)^2)^{1/2}]. \quad (12)$$

With the help of the scattering matrix, we can now write the transmission matrix:

$$\begin{bmatrix} a(0) \\ b(0) \end{bmatrix} = \begin{bmatrix} \mathcal{J}_{11} & \mathcal{J}_{12} \\ \mathcal{J}_{21} & \mathcal{J}_{22} \end{bmatrix} \begin{bmatrix} a(L) \\ b(L) \end{bmatrix}, \quad (13)$$

where

$$\mathcal{J}_{11} = \bar{\mathcal{J}}_{22} = \frac{\Delta\beta \sinh(sl) + is \cosh(sl)}{is} \exp(-i\beta_0 l), \quad (14)$$

$$\mathcal{J}_{12} = \bar{\mathcal{J}}_{21} = \frac{k \sinh(sl)}{is} \exp(i\beta_0 l). \quad (15)$$

The  $\mathcal{J}$  matrix in Eq. (13) links the left-side vector  $[a(0), b(0)]$  to the right-side vector  $[a(L), b(L)]$ . We can thus use this matrix to calculate the reflectivity:

$$R_j = \left( \left| \frac{\mathcal{J}_{11}}{\mathcal{J}_{22}} \right| \right)^2. \quad (16)$$

The advantage of this approach is that it lets us divide the grating into uniform sections, each with its own set of parameters. The total grating is simply the concatenation of these individual gratings. If we consider a uniform grating to be made up of  $m$  smaller gratings of lengths  $l_1, l_2, \dots, l_m$ , respectively ( $L = l_1 + l_2 + \dots + l_m$ ), it can be shown that

$$[\mathcal{J}_L] = [\mathcal{J}_{l_1}] [\mathcal{J}_{l_2}] \dots [\mathcal{J}_{l_m}], \quad (17)$$

where  $[\mathcal{J}_L]$  is the  $\mathcal{J}$  matrix for the whole grating and  $[\mathcal{J}_{l_i}]$  is the  $\mathcal{J}$  matrix for each section  $i$  ( $i = 1, 2, \dots, m$ ).

When the grating is divided into sections, the length of each section must contain an integer  $l$  amount of periods; i.e.,  $l_i = N_i \Lambda$ , where  $N_i$  ( $i = 1, 2, \dots, m$ ) are integers. An integer number of periods is needed to ensure coupling between sections by conserving the phase relation at the resonance wavelength.

### 4. IMPLEMENTATION

We use the genetic algorithm in conjunction with an  $F$ -matrix formalism to calculate the values of the reflected spectrum. For each individual, its values of  $(L, \Lambda, \Delta n)$  are used as parameters for the  $F$ -matrix calculation. The reflected spectrum is then computed, and the value obtained is compared with the desired value for each wavelength. The sum of all errors (for each wavelength) gives us the individual's performance.

**Table 1. Optimized Parameters of the Genetic Algorithm**

Weight ( $a$ )	1.1
Mutation operator ( $B$ )	2.0
Probability of reproduction ( $p_c$ )	45%
Probability of mutation ( $p_m$ )	15%
Number of generations	300
Population ( $\eta$ )	75

The parameters used in the genetic algorithm (probability of crossover, number of generations, etc.) must be optimized. These parameters are first set to standard values and then tweaked to produce the best performance. Once these optimal parameters are set, it is no longer necessary to modify them. The optimized parameters, in this case, are given in Table 1. The parameters in Table 1 are close to the expected values that are normally used for genetic algorithms, except for three parameters: the probability of reproduction, the probability of mutation, and the population size.

The probability of reproduction is usually approximately 80–95%. In this case, a low probability was needed to ensure that the population did not converge too quickly.<sup>19</sup> With a higher value, the population converged to a local minimum too rapidly and lost information that is important for obtaining an absolute minimum. We sped up convergence to 300 generations from 1200 by reducing the probability of reproduction from the typical 85% to 45%.

The probability of mutation will appear to be extremely high to those persons who are used to working with binary-coded genetic algorithms. However, as was noted by Michalewicz,<sup>18</sup> higher mutation rates are sometimes used in real-coded algorithms because mutations and crossovers affect the whole structure, not just single bits.

The size of the population is, as a rule of thumb, usually six or seven times the number of variables. This means that a population of 20 to 25 individuals should be satisfactory for solution of a problem in which ( $L$ ,  $\Lambda$ ,  $\Delta n$ ) are the variables. However, the highly nonlinear nature of the problem required a much bigger population, the stated value of 75 being the optimum.

## 5. RESULTS

We generated the spectra of Bragg gratings of different kinds and then used the genetic algorithm to find the parameters that would produce such spectra. For a uniform Bragg grating, with parameters  $L = 10.0$  mm,  $\Lambda = 534.0$  nm, and  $\Delta n = 1.1 \times 10^{-4}$ , the computed and real values are virtually the same, i.e.,  $L = 10.0$  mm,  $\Lambda = 534.0$  nm, and  $\Delta n = 1.1 \times 10^{-4}$  for both. Errors are not given here because the values are the same to nine decimal places. The objective function (the sum of all errors) has a value of  $4.0476 \times 10^{-10}$  after 300 iterations. A comparison of the spectra will show that the two (desired and computed values) are identical; see Fig. 2.

For chirped gratings, we must separate the grating into sections, as we are using an  $F$ -matrix formalism. Each section has a constant period ( $\Lambda$ ), length ( $L$ ), and index variation ( $\Delta n$ ), but each section is different from the oth-

ers, simulating the chirp. Each section must contain an integer number of periods.<sup>18</sup> A linear chirp function was used in calculation of the desired spectrum (calculated with Runge–Kutta integration).

Results obtained in this case, with 32 sections for calculations and a chirp of 0.02%, are listed in Table 2 and show that the parameters are basically the same. A visual confirmation is given by Fig. 3. An example of the convergence of the genetic algorithm is shown in Fig. 4 for three different trials. Tests with other chirp gratings gave similar results, although, for 0.01% chirp, good results were obtained with only eight sections.

For apodized gratings, the procedure was the same as that used for chirped gratings. In this case we used a squared cosine function to weigh the value of the index difference along the grating. The function that we used to approximate the apodization on each section is

$$\text{apo} = \frac{1}{2} \left\{ \cos \left[ \frac{\pi(k-1-0.5N_s)}{N_s} \right]^2 + \cos \left[ \frac{\pi(k-0.5N_s)}{N_s} \right]^2 \right\}, \quad (18)$$

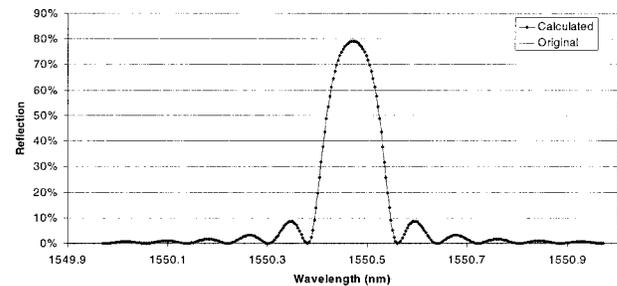
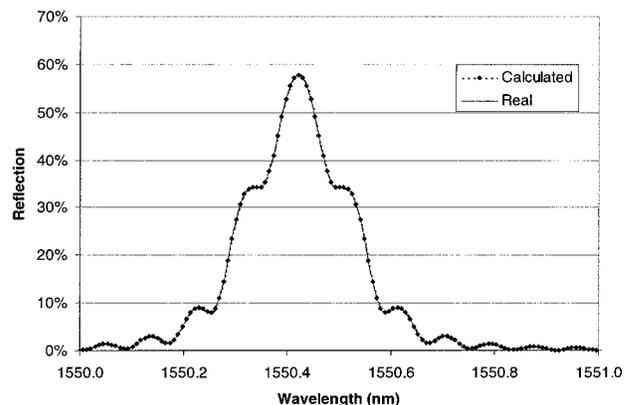


Fig. 2. Fitted and original spectra for a uniform grating.

**Table 2. Average Parameters of 10 Trials for the Chirped Grating and 32 Sections**

Value	$L$ (mm)	$\Lambda$ (nm)	$\Delta n$	$\xi$ (%)
Real	10.00	534.00	0.00011	0.02
Computed	9.99907	534.05	0.000110000013	0.019991
Error (%)	$9.7 \times 10^{-3}$	$9.3 \times 10^{-3}$	$1.2 \times 10^{-5}$	$4.6 \times 10^{-2}$

Fig. 3. Original and computed spectra for a chirped grating ( $\xi = 0.02\%$ ) calculated with 32 sections.

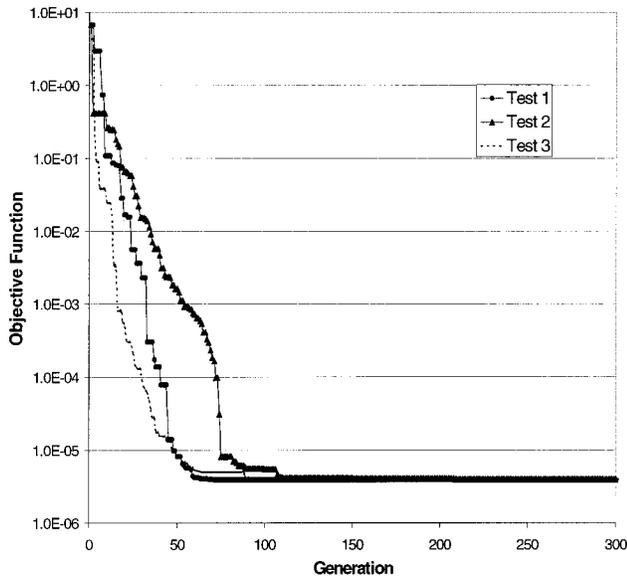


Fig. 4. Convergence of the genetic algorithm for a chirped grating (0.02%) for three different trials.

**Table 3. Results for an Apodized Grating Calculated and Eight Sections**

Value	$L$ (mm)	$\Lambda$ (nm)	$\Delta n$
Real	10.0	534.0	0.00011
Computed	9.57	534.01	0.00011636
Error (%)	4.25	$1.8 \times 10^{-3}$	5.79

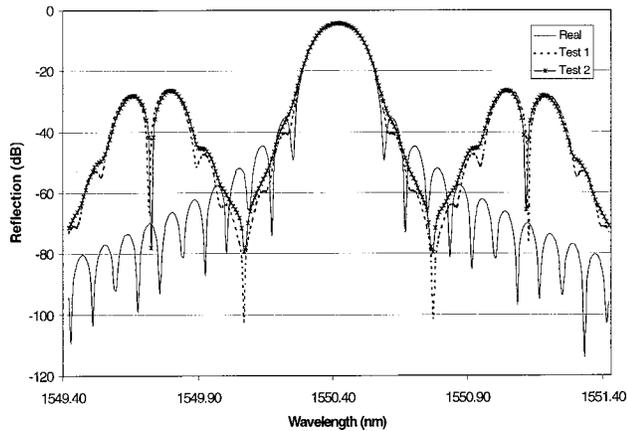


Fig. 5. Real and computed spectra for an apodized grating calculated with eight sections.

**Table 4. Results for an Apodized Grating Calculated and 100 Sections**

Value	$L$ (mm)	$\Lambda$ (nm)	$\Delta n$
Real	10.0	534.0	0.00011
Computed	10.04	534.0	0.00011
Error (%)	$3.9 \times 10^{-1}$	$6.4 \times 10^{-4}$	$6.4 \times 10^{-2}$

where  $k$  is the section number and  $N_s$  is the total number of sections. The function is minimal at the borders of the grating and maximum (=1) at the center.

Results for the apodized grating calculated with eight sections are listed in Table 3. The results obtained are not precise, with a 4% error in  $L$  and 6% error in  $\Delta n$ , as one can easily see from Fig. 5, where the central lobe is well fitted but the sidebands are not. If we increase the number of sections to 100, for example, the results are much better. Table 4 contains these new results, and Fig. 6 shows the difference between desired and computed spectra. Figure 7 shows an example of the convergence of the genetic algorithm for two different trials.

Increasing the number of sections, as shown, gives increased precision but also increases computation time. The optimum number of sections that gives the best results at an increased speed was found to be 50. Using more than 50 sections did not increase the precision of the solution, nor did it improve convergence. A minimum value of the objective function was attained after approximately 50 generations, regardless of the number of sections used.

If we apply the method discussed here to the design of a more real-world application, results obtained are similar. For example, we wish to design a bandpass filter with the following requirements:

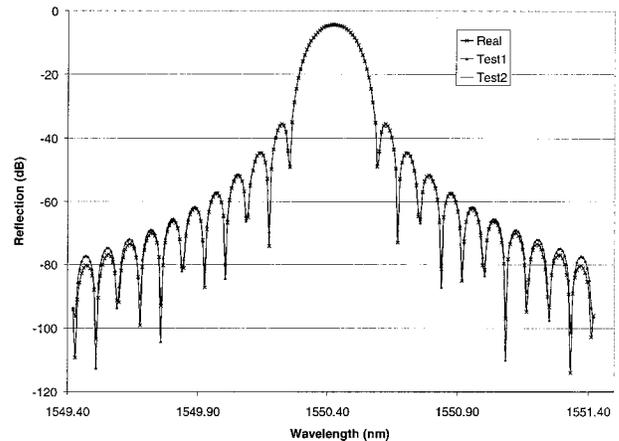


Fig. 6. Real and computed spectra for an apodized grating calculated with 100 sections.

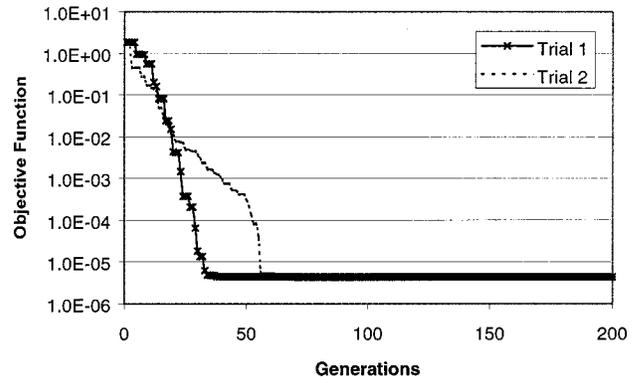


Fig. 7. Convergence of the genetic algorithm for an apodized grating calculated with 100 sections.

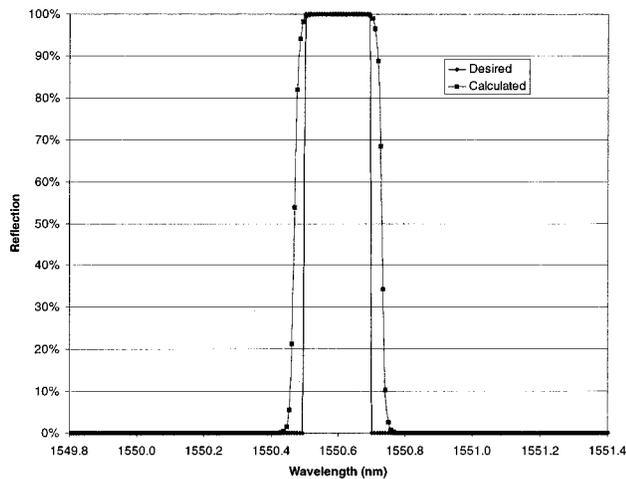


Fig. 8. Desired and calculated spectra for the design of a band-pass filter. Parameters obtained are  $L = 32$  mm,  $\Lambda = 534.01$ , and  $\Delta n = 3.8 \times 10^{-4}$ .

$$R_{\lambda} = \begin{cases} 1 & 1550.5 \text{ nm} < \lambda < 1550.7 \text{ nm} \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

The calculated reflection spectra will depend on the weighting of the objective function. We used a  $\cos^2$  apodization function to create the filter. A weighting coefficient was used; its value depended on which part of the spectrum was deemed more important. In this case, as an example, we wished to have a maximum of reflection in the passband. In the case of Fig. 8, the reflected values calculated from 1550.5 to 1550.7 nm are multiplied by 15. Depending on the weighting coefficient, the reflected spectrum can be thinner or wider. Should the cut-off wavelength be more important, a simple modification of the objective function will yield a filter with a lower maximum reflection but a sharper cutoff.

## 6. CONCLUSION

Results obtained have shown that one can use the genetic algorithm to determine the parameters of Bragg gratings by use of only the reflection spectrum information. Approximation with the  $F$ -matrix formalism requires one to divide the grating into sections. An increase in the number of sections increases the precision but also increases the computing time. The genetic algorithm is easily modifiable to work with more-complex spectra and design parameters. It can easily be modified to be used to design a bandpass filter, although that technique is not so precise as other proposed methods.

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## REFERENCES

1. B. Ortega, L. Dong, and L. Reekie, "All-fiber optical add-drop multiplexer based on a selective fused coupler and a single fiber Bragg grating," *Appl. Opt.* **37**, 7712–7717 (1998).
2. S. Thériault, K. O. Hill, F. Bilodeau, D. C. Johnson, J. Albert, G. Drouin, and A. Béliveau, "High-g accelerometer based on in-fiber Bragg grating," *Opt. Rev.* **4**, 145–147 (1998).
3. L. Y. Lo, "Using in-fiber Bragg grating sensors for measuring axial strain and temperature simultaneously on surfaces of structures," *Opt. Eng.* **37**, 2272–2276 (1998).
4. J. A. R. Williams and I. Bennion, "Applications of fiber gratings in microwave photonics," Photonics Research Group, Aston University, <http://benedick.aston.ac.uk/Photonics/publications/MR97/> (1997).
5. K. A. Winick and J. E. Roman, "Design of corrugated waveguide filters by Fourier transform techniques," *IEEE J. Quantum Electron.* **26**, 1918–1929 (1990).
6. G. H. Song and S. Y. Shin, "Design of corrugated waveguide filters by the Gel'fand–Levitan–Marchenko inverse-scattering method," *J. Opt. Soc. Am. A* **2**, 1905–1915 (1985).
7. E. Peral, J. Capmany, and J. Martí, "Iterative solution to the Gel'fand–Levitan–Marchenko coupled equations and application to synthesis of fiber gratings," *IEEE J. Quantum Electron.* **32**, 2078–2084 (1996).
8. R. Feced, M. N. Zervas, and M. A. Muriel, "An efficient inverse scattering algorithm for the design of nonuniform fiber Bragg gratings," *IEEE J. Quantum Electron.* **35**, 1105–1115 (1999).
9. J. Skaar and K. M. Risvik, "A genetic algorithm for the inverse problem in synthesis of fiber gratings," *J. Lightwave Technol.* **16**, 1928–1932 (1998).
10. S. Huang, M. LeBlanc, M. N. Ohn, and R. M. Measures, "Bragg intragrating structural sensing," *Appl. Opt.* **34**, 5003–5009 (1995).
11. J. H. Holland, *Adaptation in Natural and Artificial Systems* (U. Michigan Press, Ann Arbor, Mich., 1975).
12. R. Boudreau and N. Turkkan, "Solving the forward kinematics of parallel manipulators with a genetic algorithm," *J. Rob. Syst.* **13**, 111–125 (1996).
13. A. Minato and N. Sugimoto, "Design of a four-element, hollow-cube corner retroreflector for satellites by use of a genetic algorithm," *Appl. Opt.* **37**, 438–442 (1998).
14. G. Cormier and R. Boudreau, "Genetic algorithm for ellipsometric data inversion of absorbing layers," *J. Opt. Soc. Am. A* **17**, 129–134 (2000).
15. A. H. Wright, "Genetic algorithms for real parameter optimization," in *Foundations of Genetic Algorithms*, G. J. E. Rawlins, ed. (Morgan Kaufman, San Mateo, Calif., 1991), pp. 205–218.
16. S. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning* (Addison-Wesley, Reading, Mass., 1989).
17. L. Davis, *Handbook of Genetic Algorithms* (Van Nostrand Reinhold, New York, 1991).
18. Z. Michalewicz, *Genetic Algorithms* (Springer-Verlag, New York, 1992).
19. T. Kuo and S.-H. Hwang, "Using disruptive selection to maintain diversity in genetic algorithms," *Appl. Intel.* **7**, 257–267 (1997).