Characterizing spatial variability of a clay by geostatistics
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Abstract: This paper presents a characterization of the variability of a lightly overconsolidated and highly sensitive clay deposit located near Saint-Hilaire, 50 km east of Montréal. The geotechnical investigation consisted of in situ and laboratory tests. The variability of the in situ test results is the subject of this paper. The working hypothesis assumes that piezocene and vane test results may be modelled by a random function. This is done on the basis of a geostatistical approach. In situ vane and piezocene tests are found to increase with depth following a linear trend. This is a nonstationary problem and inference of the autocorrelation function must be made through the estimation of a generalized covariance. Results for both types of tests give the same shape of generalized covariance. Measurements made with both testing devices yield the same 2 m autocorrelation distance but the standard deviations are different. The standard deviations for the piezocene cone resistance, pore pressure behind the cone tip, and sleeve friction are 74, 34, and 2.1 kPa, respectively. Vane measurements have a standard deviation of 4.9 kPa. Results are also presented for the estimation of the vertical linear trend and for the statistical distributions of fluctuations.

Key words: sensitive clay, spatial variability, stochastic representation, geostatistics, piezocene testing, vane testing.


Mots clés: argile sensible, variabilité spatiale, représentation stochastic, géostatistique, essai au piezocene, essai au scissomètre.

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Introduction
In a soil investigation, the geotechnical engineer looks for answers to a specific number of questions. The nature of these questions depends on the type of project and its size. In general, an overall picture of subsurface conditions is first searched for. The geotechnical engineer tries at this point to define a typical stratigraphic subsoil section by using boreholes, test pits, etc. Laterly, in situ tests like the piezocone are also used for stratigraphic characterization purposes (Robertson 1990). Knowing what type of layers are involved helps the engineer to better define the problem. The necessary data to solve the problem must then be sought.

The next step in the geotechnical investigation is the interpretation of the results. The engineer must choose the proper parameters as input to the analysis. This decision of selecting the right parameters is based on experience,
knowledge, and judgment. The traditional approach often assumes conservative or pessimistic values in calculating safety factors. The engineer usually has a feel for the variability and will often order more investigation to better characterize the upper and lower limits. However, geo-
technical engineers generally lack the tools to quantify this variability and incorporate it in an analysis.

The site of Saint-Hilaire, subject of this study, displays an important variability problem (Fig. 1). Characterizing this variability is of great importance, since this would allow evaluation of the odds of a slope-failure scenario. Such a characterization may be done using a geostatistical approach, which quantifies not only the data dispersion, but also the spatial dependence of the variability. Basically, geostatistics analyze the natural random character of a soil with a tool called the spatial covariance (or variogram). This covariance is a function that contains three elements of information. First, all of the covariance is an appreciation of the dispersion of the parameters, which equates to the variance (i.e., the covariance at zero lag). Secondly, it supplies an autocorrelation distance that represents the radius of influence of a measurement made at a given point. Thirdly, it provides the type of variability that indicates how values fluctuate in space.

Stochastic modelling of clay variability in space is becoming more and more popular among researchers. Following the tracks of Aouak and Giroux (1982), Ravi (1992) used a space-series analysis to study vane profiles following a linear or quadratic trend, and Soulie et al. (1990) used a geostatistical model to study horizontal and vertical variability in a deposit where vane strengths fluctuated around a constant mathematical expectation. In this study, we wish to show that a geostatistical approach can also adequately model the spatial parameters measured by vane- and piezocene profiles. This is the presence of a vertical trend. This paper presents quantified variability results for the intact clay layer. These include the experimental spatial autocorrelation function (i.e., generalized covariance) and the statistical distribution of fluctuations around the local vertical trend. Finally, the validity of the model is cross-checked with in situ data.

Site description

The site is located 50 km east of Montreal, near the village of Saint-Hilaire. Figure 2 presents a typical geotechnical profile. This profile shows that the site is formed of three distinct soil layers. From the top, a thin layer of fine uniform beach sand is followed by a thick 32 cm layer of clay. This clay is stiff and weathered in the top 2-3 m. Under the crust lies a sensitive, lightly overconsolidated clay. Finally, the boreholes were confined on a confined aquifer composed of a permeable glacial till (silty sand with some gravel). A small underdrained condition exists in the clay layer. Laliberté et al. (1988) describe the results of a test examination made on this site.

The geotechnical investigation includes in situ and laboratory tests. Of the in situ tests, 16 profiles were performed with the piezocene, and 27 with the vane. Profile depths are 16.6 m for the piezocene and 15.5 m for the vane. Horizontal spacing between profiles is typically 10 m, with a vane sounding every 3 m to each piezocene. On the vertical axis, readings are spaced by 0.5 m for the vane and by less than 0.1 cm for the piezocene. The site plan shows a site plan. The in situ test data base consists of 10,682 piezocene measurements and 689 vane results. Laboratory tests were also performed (McCallach 1987) but are outside the scope of this paper.

Geostatistical site characterization

The following discussion is subdivided according to the two types of measurements. Piezocene vertical variabil-
ity is studied first, followed by vane test results. Finally, results obtained from both types of devices are compared.

Piezocene variability

The first step is to evaluate the absence or presence of a trend (i.e., stationarity or nonstationarity) in the measure-
ments when following the vertical direction. This can be studied in two ways. One way is a simple visual exami-
nation of the piezocene results as a function of depth. Fig-
ure 2, which illustrates one of the 15 piezocene profiles, shows the natural tendency of the cone resistance (r_2) to increase with depth. A better way is to calculate the follow-

\[ \Delta h_j = \frac{1}{N_j} \sum [r(z_j + h_j) - r(z_j)] \]

where

\[ \Delta h_j \]

is mean first-order increment of data vertically spaced by \( h \), and \( r(z_j) \) is measured parameter at depth \( z \).

Note that this increment filters constants. If measurements
fluctuate around a constant average value (i.e., are stationary), mean increments will tend toward zero, regardless of lag h. But if measurements follow a trend (nonstationary), increments will tend to increase with lag h. Figure 4 presents mean first-order increments for the three sets of data obtained from the piezocone. It eloquently shows that measurements do not fluctuate around a constant. Mean first-order increments increase following a straight line, indicating a linear trend. In other words, piezocene measurements fluctuate around a trend that increases linearly with depth.

Classical geostatistics is made for stationary problems. In these problems, measurements fluctuate around a constant mathematical expectation. When considering a trend, a more general theory must be used. Matheron (1973) called this theory intrinsic random functions of order k (or IRRF-k for short). Deffner (1976) gives a more practical description, and his paper is recommended for those readers who wish more detailed explanations. Since the object of this paper is not to discuss this theory in depth, a general description will be made using classical geostatistics as a working basis.

In classical geostatistics, in situ parameters are regionalized variables (RVs). Regionalization means that the variable is intrinsically related to its spatial position. Since this variable cannot be known everywhere, and it cannot be easily represented by a mathematical function, it is convenient to imagine the deposit as a particular outcome of a random process. This random process must take into account the regionalized character of the variable. Therefore not any type of random process is acceptable. Stationary random functions (SRF) have this property. Let us then represent an in situ parameter $V(x)$ by the following:

$$V(x) = m(x) + R(x)$$

where $m(x)$ is mathematical expectation (constant for a stationary process) at a given position $x$ in space, and $R(x)$ is zero mean stationary random function representing fluctuations in function of position $x$ in space.

When realizations of $R(x)$ are closely spaced, data take neighbouring values, indicating close interdependence. When realizations are far apart, data tend to become

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**Fig. 2. Typical geological profile.**

**Fig. 3. Plan of site near Saint-Hilaire.**

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independent from one another. The lag where data start to be completely independent is called the autocorrelation distance $a$ or the range of the covariance $C(h)$. This covariance is defined as

$$C(h) = E[(V(x + h) - m)(V(x) - m)] = E[V(x + h)V(x)] - m^2$$

For a natural process, this spatial covariance is a priori unknown. It has to be estimated. To do so, the preferred tool is the variogram $2\gamma(h)$, since by definition it filters constants, and in stationary processes the mean $m$ of $\{V\}$ is a constant. Filtering the mathematical expectation implies that fluctuation variances are measured without bias. Mathematically, the variogram is written as

$$2\gamma(h) = Var[V(x + h) - V(x)] = E[(V(x + h) - V(x))^2]$$

When a finite variance exists, the following relation links the variogram to the covariance $C(h)$:

$$C(h) = C(0) - 2\gamma(h)$$

where $C(0)$ is the covariance at zero lag and corresponds to the a priori variance of the fluctuation.

In the case of a nonstationary process, the mean value $m(x)$ (eq. (2)) is no more a constant. This mean value can be expressed, at least locally, by polynomials of degree $k$ with unknown coefficients. To characterize intrinsically $V(x)$, we have to manipulate expressions of $V(x)$ which are not dependent on these coefficients.

For stationary processes, it is sufficient to use increments of order zero, namely $[V(x + h) - V(x)]$, which filter out the unknown constant mean. For polynomials of order $k$, increments of order $k$ are used, since they filter
out the unknown polynomial trend. The variance of these increments can be expressed by a generalized covariance (Matheron 1973). For a stationary phenomenon, the semi-
varianogram γ (half the variogram) is a generalized covari-
ance of order zero.

A regionalized variable may be considered as the sum of a stationary random function and a deterministic trend
when the generalized covariance has a sill (i.e., a hori-
zontal threshold). In such a case the relationship between
the generalized covariance K(h) and the covariance C(h) is
as follows:

\[ \text{C}(h) = \text{K}(h) - \text{K}(0) \]

and where \( \text{K}(0) \) represents the sill value of the general-
ized covariance. As a consequence, the relationship between the
generalized covariance and the a priori variance \( \text{C}(0) \) is
as follows:

\[ \text{C}(0) = \text{K}(0) - \text{K}(0) \]

For a zero-valued \( \text{K}(0) \), the a priori variance is then simply
the magnitude of the generalized covariance sill.

The experimental generalized covariances of order 1 for
the piezocene measurements have been evaluated (Chiasson 1993) and are presented in Fig. 5. Note that each general-
ized covariance is scaled by a kaisp so as to fit in the same
graph. All three measurements have an experimental gen-
eralized covariance that admits a sill. As discussed in the
preceding paragraph, the existence of such a sill indicates
that the random process may be modelled by the sum of
a stationary random fluctuation with a vertical linear trend.
The fluctuations part has a finite a priori variance \( \text{C}(0) \),
which is represented by the sill \( \text{K}(0) \) of the generalized covariance \( \text{K}(h) \). From Fig. 5, a priori variances \( \text{C}(0) \) for the
piezocene cone resistance \( q_c \), pore pressure behind the cone
tip \( u_c \), and sleeve friction \( f_s \) are 5400, 1180, and 4.6 kPa²,
respectively. In terms of standard deviations (not shown in
Figure 5), the results are 74, 34, and of 2.1 kPa, respec-
tively. Data from Fig. 5 also show that all three piezocene
measurements have experimental generalized covariances
with a 2.0 m autocorrelation distance which pass through
zero and behave in a linear fashion near the origin. This 2
m autocorrelation distance indicates that data spaced by
more than this distance are not correlated. This means that
a measurement has no influence on the fluctuation part of

\[ m_c = A_0 + A_1 z \]

where \( A_0 \) is the true intercept of the vertical trend, and
\( A_1 \) is the true slope.

A priori, this vertical trend is unknown. This is why
we use the term estimate, since one can only do so with
the in situ measurements. For details on how this is done,
the reader should consult Matheron (1971). Figure 6 shows coefficient estimates for cone resistance \( q_c \) plotted in
\( \{ A_0, A_1 \} \) space for all piezocene profiles. It shows con-
siderable scatter. Observed \( q_c \) trends have slopes as high
as 56 kPa/m, or as low as 21 kPa/m, although slopes with
high values are associated with low intercepts. Intercepts are
as low as 51 kPa and as high as 585 kPa. This last figure
also shows considerable scatter and one must ask if this
scatter is solely due to the estimation error. Coefficient
estimates are obtained knowing at this step and confi-
dence intervals for each coefficient may be drawn. To
do so, one must assume some probability law. Such con-
fidence intervals for cone resistance \( q_c \) trend coefficients
using a normal law are shown in Fig. 7. This figure clearly
shows that estimation errors do not totally explain scat-
ter. One can observe that a theoretical 95% interval only
includes 56% of intercept coefficients or 81% of slope
values. This scatter is inconsistent with a stochastic rep-
resentation based on a pure vertical site trend.

Another possibility is to consider a representation based
on a vertical trend that is dependent on its position in the
horizontal plane. In other words, the vertical trend is in
itself a regionalized variable. This regionalized character
appears obvious in Fig. 6. Vertical trends on the north side of the excavation tend to have high slope values with low intercepts, whereas profiles on the south side generally display low slopes with high intercepts. Profiles on the west side tend to be intermediate in value. These observations indicate that a correlation exists between profile position in the horizontal plane and vertical trend values.

The preceding paragraphs indicate that the mathematical expectation t exp of [2] is a function of depth and position in the horizontal plane. As a consequence, the variogram cannot be used for a horizontal-variability study. As for a generalized covariance estimation, data can only be used if thek form groups of at least three sampling points on the same straight line. For the piezocene, the site plan (Fig. 3) shows only three sampling axes in the horizontal plane. The maximum lag that may be investigated is 60 m. Furthermore, a geostatistical rule (Journel and Huijbregts 1978) considers an autocorrelation function valid when the estimation is based on a minimum of 50 data groups. Since data were only independent at a vertical lag of 2 m, only eight piezocene measurements per profile (16.63 m length per

2 m range) are independent. This implies that the rule is only respected for a lag of 40 or more. This is why a horizontal-variability study is impossible with the available data. A sampling grid better adapted for such a study would be regular (i.e., square) and would define the location of approximately 50 profiles. Since a horizontal variability is unfeasible at this stage, a representation based on a random trend is proposed. Outcomes of this random trend should be limited to the space occupied by observed piezocene values such as those of Fig. 6.

Adopting a stochastic representation permits us to calculate residues. These residues approximate realizations of the fluctuation component \( R(s) \) of the total profile. Residues are formed when subtracting the local trend from the measured profile. It is to be noted that, in light of the earlier discussion, the local trend is used instead of a global mean trend. Results for means and variances of piezocene residues are shown in Table 1. Mean values are close to zero, indicating a successful unbiased trend estimation. Estimated variances \( \sigma^2 \) are close to theoretical covariance values. However, \( \chi^2(0) \) is not a unit ratio. This is in compliance with theory. Since parameters are spatially autocorrelated, variance from a finite domain will give a somewhat smaller result than the global a priori variance \( \sigma^2(0) \). For the adopted spherical model, the theoretical ratio is 0.89. Values presented in Table 1 are quite close to this theoretical result. In fact, it could be more likely explained by estimation error, but this would need further estimation of fourth-order moments. Congruence of the theoretical and experimental ratios does not justify such work.

The last result is the statistical distribution of the piezocene residues. Figure 8 gives the statistical distribution for cone resistance \( q_c \) residues. The induced pore pressure \( u \) and corrected slope friction \( j_s \) yield the same shape frequency distribution. It is a rather symmetrical distribution, but somewhat tighter around the mean than the normal curve, although the 95% interval is at approximately the same position. This fact is important in view of the criteria used when choosing a representation for the vertical trend. Statistical tests such as the \( \chi^2 \) test or Kolmogorov-Smirnov test confirm that the law is not normal. The deviation from the experimental distribution remains quite small though, so it is entirely satisfactory to use a normal law process.

Vane test variability

Similar to piezocene test results, vane strength \( q_v \) measurements show a strong tendency to increase with depth. This is well illustrated in Fig. 1, and mean increments

<table>
<thead>
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<th>Table 1. Mean and variance of piezocene local vertical trend residues.</th>
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<tr>
<td><strong>Mean residue</strong> (kPa)</td>
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<tr>
<td>------------------------</td>
</tr>
<tr>
<td>Cone resistance ( q_c )</td>
</tr>
<tr>
<td>Pore pressure ( u )</td>
</tr>
<tr>
<td>Sleeve friction ( j_s )</td>
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yield the same linear trend as in Fig. 4. This nonstationarity implies that RBF k = 1 theory must again be used to model the phenomena. In this theory, which was discussed earlier, the autocorrelation function is evaluated by means of a generalized covariance.

Figure 9 presents experimental vertical generalized covariances $k(h)$ for two sets of data. The first set is formed by vane profiles that are adjacent to a piezoecone. The second includes all vane tests performed on the site. Again, a 2 m range is evident, but sill values are different. The adjacent set has an a priori variance of 0.5 of 27 kPa, which is higher than the 24 kPa value of the second set. This apparent inconsistency is explained by estimation error. Since the largest set has a lower estimation error, the value of 24 kPa must be closer to the a priori variance of the site. The same covariance function used for the piezoecone, that is, the spherical model with a 2 m range, fits the experimental generalized covariance results of the vane closely.

Trend estimates are calculated using this model. These trend estimates are illustrated in Fig. 10. Again, one can identify a relationship between profile position in the horizontal plane and trend parameters. In the same way as for piezoecone results, scatter of estimated vane trends is not caused by estimation variance of a unique site trend. Further investigation of horizontal variability is not possible, since the sampling grid is not favourable to such a study.

Since the vertical trend is not unique, it is assumed to be random. This randomness should not be without limits. Random outcomes should not be too far away from observed ones. Figure 10 shows a trapezoidal domain that is constructed in a way to include all observed trends of the set of vanes adjacent to a piezoecone. This same domain includes almost every trend of all the vane tests. We propose to use this trapezoidal as a limiting domain to the randomness of the trend. The adopted representation is then a random trend with coefficients limited to the trapezoid of Fig. 10. The coefficients of the trend are generated by a uniform law. In this way, every point included in the trapezoid is equiprobable.

The complete representation for vane measurements is then a random trend added to a fluctuation that is a random function having a spherical covariance of 2 m range and 24 kPa sill. This representation can be used to simulate vane profiles, which will have the same characteristics as true vane measurements.

Calculating residuals between the linear trend of each profile and the experimental measurements leads to a statistical distribution of vane fluctuations. The result for all vane measurements is shown in Fig. 11. This distribution is not normal, but it is well approximated by this law, since discrepancies are small.

To better validate the adopted representation, a cross-check is performed between theoretical results and observed
measurements. It is to be remembered that vane profiles were performed each 0.5 m of depth. Let us subdivide each profile into two sets of measurements spaced by 1 m. The first set is used as conditioning points. The second set is considered as realizations at mid-distance between the conditioning points. In this way, one can form an estimate of the conditional variance and compare it with its theoretical value. In geostatistics, local estimation is performed by means of the technique of kriging. Using the conditioning points and the generalized covariance \( K(h) \) model of the regionalized variable \( z(x) \), a best linear estimator \( \hat{z}_k(x) \) (i.e., a universal kriging estimator) is determined at the realization set coordinates \( x \). A residue is then formed between the kriged estimate and the realization represented by the second set. Since kriging is an unbiased estimator of the conditional expectation, residues should have a zero mean. This is the case and acts as a first confirmation of the validity of the representation. Finally, the squared residues yield a value of 8.85 kPa\(^2\) for the conditional variance. This conditional variance is an experimental outcome of the kriging variance. The theoretical value of this kriging variance, which is based on the adopted representation, is 9.20 kPa\(^2\). It is to be noted that the adopted representation is a random trend combined with a random function (i.e., the fluctuation) having a spherical covariance with a 2 m range and a 24 kPa\(^2\) sill. The slight discrepancy between both values is, based on a \( \chi^2 \) test, insignificant and explains itself by estimation error. Finally, to conclude this study of vane variability, we propose to look at a simulated vane profile, versus true measurements. Figure 12c shows a continuous simulation of a vane profile based on the adopted representation. Since true measurements cannot be obtained in a continuous manner, data points were sampled each 9.5 m from the simulated profile. This sampled profile is compared with the true vane profile in Fig. 12b. Results are similar that one cannot discriminate between them prior without prior knowledge. This is a qualitative demonstration that the geostatistical model adopted here is quite representative of the in situ variability.

1 Universal kriging of order 1 is used when in the presence of an unknown linear trend. The estimation that has to be determined is of the form

\[
\hat{z}_k(x) = \sum_{i=1}^{n} \lambda_i z_i(x)
\]

On the right-hand side are the \( n \) conditioning points \( z_i(x) \) and the weighting factors \( \lambda_i \). The weighting factors are such that the estimator is unbiased and the variance of estimation is minimum. These factors are solutions to the following system of equations:

\[
\sum_{j=1}^{n} K(x_j - x_i) \lambda_j = y_{i0} + y_{i1} x_i, \quad i = 1, \ldots, n
\]

and

\[
\sum_{j=1}^{n} \lambda_j = 1
\]

where \( y_{i0} \) and \( y_{i1} \) are Lagrange functions. The variance of estimation, or kriging variance, is

\[
\text{Var}(\hat{z}(x)) = \sum_{i=1}^{n} \left( y_{i0}^2 \right) - \sum_{i=1}^{n} K(x - x_i)
\]

Discussion of results

It is interesting to note that the adopted covariance model for vertical variability is for both testing devices, a spherical covariance with a 2 m range. Except for their sill, results seem to show that both testing devices give the same structural information. To better investigate this similarity in covariance model, let us standardize experimental generalized covariances \( K(h) \) by their respective a priori variance or sill. Figure 13 shows kriged and vane results of the generalized covariance in such a standardized fashion. It shows tightly overlapped data points. This result has interesting implications. For one thing, it adds robustness to the chosen model, since two different types of devices yielded the same autocorrelation function. For another, it has consequences on geotechnical investigations, because one can use piezoecone measurements to assess vane variability, and vice versa. This is very interesting, and one must question if this result may be generalized to other types of undrained strength measurements such as laboratory unconfined compression. The fact that data points are so tightly overlapped indicates that both testing devices, although quite different in loading mode, give the same structural information.

The observed experimental generalized covariances have a sill, linear behaviour near the origin, and a 2 m auto-correlation distance and lack a nugget effect. What does
Fig. 13. Vertical experimental generalized covariances $K(h)$ for piezocoon and vane test measurements standardized by their respective variances $C(0)$.

This means exactly? First of all, let us look at the presence of a sill. This sill indicates that the phenomena has a finite variance. In other words, values closely fluctuate around an expectation, i.e., the trend, and extreme values have very low probability. Second, the linear behaviour near the origin indicates that vane or piezocoon profiles are very erratic functions. This is well illustrated by the continuous simulation presented in Fig. 12c or by the piezocoon profile of Fig. 2. Thirdly, there is the absence of a nugget effect. This absence indicates that no random measurement error was made during testing. This result is in agreement with another study on vane variability by Soulé et al. (1990) which came to the same conclusion. Note that this result does not discard a possible systematic error. The absence of a nugget effect also indicates that the random function representing vane or piezocoon measurements is a continuous one. Fourthly, there is the 2 m vertical autocorrelation distance. This result is in the same order of magnitude to those reported by Soulé et al. (1990) or Chowdhury (1984).

As for the vertical trend, it is found to be linear, but it is not unique to the site. It rather seems to be related to its position in the horizontal plane. Unfortunately the sampling grid is unfavorable to a more detailed study of this subject. This is why a representation based on a random trend is adopted. Realizations of these random trends are limited to a space of observed values.

The similarity between statistical parameters does not stop at the autocorrelation function, since the same shape of standardized statistical distribution is also observed for both testing devices (Fig. 14). This statistical distribution is not normal, although this law can be used for practical purposes, since discrepancies are small.

In the previously mentioned study of Soulé et al. (1990), a clay deposit from the James Bay area in Quebec was modeled with the help of a stationary random process. In this case, the deposit had a geometric anisotropy with autocorrelation distances in the vertical and horizontal direction of 3 and 30 m, respectively. In comparison, the Saint-Hilaire clay is found to be nonstationary in the vertical direction, although it has a 2 m vertical autocorrelation distance, which is in the same range as the preceding site. Another difference lies in the vertical trend coefficients of the Saint-Hilaire site which are found to be dependent on their position in the horizontal plane. Although it could not be thoroughly examined in this study, it appears that the horizontal variability at Saint-Hilaire may also contain a trend term. To summarize, spatial variability of clay at Saint-Hilaire is not of the same nature as at the James Bay site. As is the case for other clay properties, which may vary from one site to another, different locations may have different geological representations. This is not a problem though, since both cases were adequately modeled by the theory of geostatistics.

Conclusions

We have demonstrated that a geostatistical model can be a good representation of the variability of a clay deposit when in the presence of a vertical trend. Vane or piezocoon vertical profiles are continuous but erratic functions and fluctuate around a vertical trend. These fluctuations have a finite variance, and the vertical trend is variable over the site. Although this trend is probably related to its position in the horizontal plane, an unfavorable sampling grid does not permit a more detailed study of this subject. This is why a representation using a random trend is adopted.

Another important result of this study is that both testing devices, i.e., vane and piezocoon, yield the same spatial covariance and statistical distribution. This is a guarantee that the assessed variability, which is the adopted representation, is not dependent on the operating mode of the
Fig. 14. Standardized statistical distributions for both piezocene and vane measurements.

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References